5.5 Method of Joints

This method involves applying COE to individual joints after isolating them from the parent truss. This method is based on the principle "If the truss is in equilibrium, an isolated joint of the truss will also be in equilibrium". The following steps are involved while analysing truss by method of joints.

- Step 1: Find the reactions at the supports of the truss by applying COE to the entire truss.
- Step 2: Isolate a joint from the truss which has not more than two members with unknown force.
- Step 3: Assume that the members carry tension force. Based on this assumption show the arrows on the unknown member pointing away from the joint.
- Step 4: The forces at the joint form a concurrent force system to which we can apply two COE viz. $\Sigma F_x = 0$ and $\Sigma F_y = 0$ to find the unknown force in the members. If the value obtained is negative it implies that the assumption was incorrect and the member carries compressive force and not tensile force.
- Step 5: Mark the magnitude and nature of the force so obtained on the parent truss and now isolate another joint having not more than two members with unknown force. Follow Steps 2-5 as before and thus solve joint after joint to find forces in all the members of the truss.
- Step 6: Tabulate the results indicating the member, its force magnitude and the nature of the force.
- Ex. 5.1 Using method of joints, analyse the truss shown.

Solution:

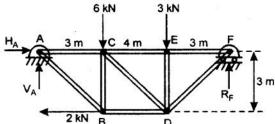
Support reactions:

Applying COE to the entire truss

$$\sum F_x = 0 \Rightarrow + \text{ ve}$$

$$H_A - 2 = 0$$

$$\therefore H_A = 2 \text{ kN} \Rightarrow$$



Isolating joint A

Joint A can be isolated since there are two unknown members AB and AC. Initially assuming the members to be in tension

Applying COE
$$\sum F_{y} = 0 \uparrow + ve$$

$$4.5 - F_{AB} \sin 45 = 0$$

$$F_{AB} = 6.36 \text{ kN}$$

$$F_{AB} = 6.36 \text{ kN (Tension)}$$

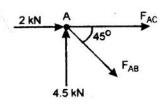
$$\sum F_{x} = 0 \rightarrow + ve$$

$$2 + F_{AC} + F_{AB} \cos 45 = 0$$

$$2 + F_{AC} + 6.36 \cos 45 = 0$$

$$F_{AC} = -6.5 \text{ kN}$$

$$F_{AC} = 6.5 \text{ kN (Compression)}$$



Isolating joint B

This joint has two unknown members BD and BC. Initially assuming them to be in tension.

Applying COE

$$\sum F_{x} = 0 \rightarrow + ve$$

$$-2 - 6.36 \cos 45 + F_{BD} = 0$$

$$F_{BD} = 6.5 \text{ kN}$$

$$F_{BD} = 6.5 \text{ kN (Tension)}$$

$$\sum F_{y} = 0 \uparrow + ve$$

$$6.36 \sin 45 + F_{BC} = 0$$

$$F_{BC} = -4.5 \text{ kN}$$

$$F_{BC} = 4.5 \text{ kN (Compression)}$$

Isolating Joint C

This joint has two unknown members CE and CD. Initially assuming them to be in tension

Applying COE

$$\sum F_{y} = 0 \uparrow + ve$$

$$4.5 - 6 - F_{CD} \sin 36.87 = 0$$

$$F_{CD} = -2.5 \text{ kN}$$

$$= 2.5 \text{ kN (Compression)}$$

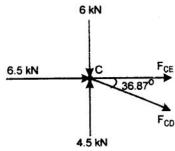
$$\sum F_{x} = 0 \rightarrow + ve$$

$$6.5 + F_{CD} \cos 36.87 + F_{CE} = 0$$

$$6.5 + (-2.5) \cos 36.87 + F_{CE} = 0$$

$$F_{CE} = -4.5 \text{ kN}$$

$$F_{CE} = 4.5 \text{ kN (Compression)}$$



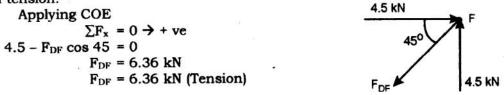
Isolating Joint E

This joint has two unknown members EF and ED. Initially assuming the members to be in tension

Applying COE $\begin{array}{c} \Sigma F_x = 0 \rightarrow + \text{ ve} \\ 4.5 + F_{EF} = 0 \\ F_{EF} = -4.5 \text{ kN} \\ F_{EF} = 4.5 \text{ kN (Compression)} \end{array}$ $\begin{array}{c} \Sigma F_y = 0 \uparrow + \text{ ve} \\ -3 - F_{ED} = 0 \\ F_{ED} = -3 \text{ kN} \\ F_{ED} = 3 \text{ kN (Compression)} \end{array}$

Isolating joint F

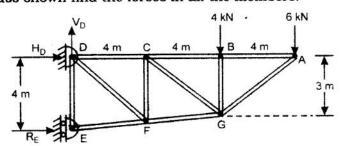
This joint has only one unknown member DF. Initially assuming the member to be in tension.



Finally tabulating the results.

Member	Force (kN)	Nature
AB	6.36	Tension
AC	6.5	Compression
BD	6.5	Tension
BC	4.5	Compression
CD	2.5	Compression
CE	4.5	Compression
EF	4.5	Compression
ED	3	Compression
DF	• 6.36	Tension

Ex. 5.2 For the truss shown find the forces in all the members.



Solution: This truss is special since it has a *cantilever end*. Cantilever end is a joint having only two members and is externally unsupported. Analysis of a truss having a cantilever end need not begin from finding the reactions. One can start solving from the cantilever end.

The cantilever end of the given truss is joint A.

5.5 Method of Joints

This method involves applying COE to individual joints after isolating them from the parent truss. This method is based on the principle "If the truss is in equilibrium, an isolated joint of the truss will also be in equilibrium". The following steps are involved while analysing truss by method of joints.

- Step 1: Find the reactions at the supports of the truss by applying COE to the entire truss.
- Step 2: Isolate a joint from the truss which has not more than two members with unknown force.
- Step 3: Assume that the members carry tension force. Based on this assumption show the arrows on the unknown member pointing away from the joint.
- Step 4: The forces at the joint form a concurrent force system to which we can apply two COE viz. $\Sigma F_x = 0$ and $\Sigma F_y = 0$ to find the unknown force in the members. If the value obtained is negative it implies that the assumption was incorrect and the member carries compressive force and not tensile force.
- Step 5: Mark the magnitude and nature of the force so obtained on the parent truss and now isolate another joint having not more than two members with unknown force. Follow Steps 2-5 as before and thus solve joint after joint to find forces in all the members of the truss.
- Step 6: Tabulate the results indicating the member, its force magnitude and the nature of the force.
- Ex. 5.1 Using method of joints, analyse the truss shown.

Solution:

Support reactions:

Applying COE to the entire truss

 $V_A - 6 - 3 + R_F = 0$ $V_A - 9 + 4.5 = 0$

$$\sum M_A = 0 \qquad + ve$$

$$-2 \times 3 - 6 \times 3 - 3 \times 7 + R_F \times 10 = 0$$

$$\therefore \qquad R_F = 4.5 \text{ kN} \uparrow$$

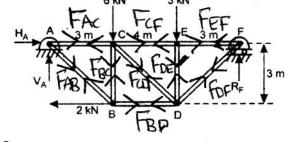
$$\sum F_x = 0 \rightarrow + ve$$

$$H_A - 2 = 0$$

$$\therefore \qquad H_A = 2 \text{ kN} \rightarrow$$

$$\sum F_y = 0 \uparrow + ve$$

 $V_A = 4.5 \text{ kN} \uparrow$



Applying COE

$$\Sigma F_{x} = 0 \rightarrow + \text{ ve}$$

$$4.5 + F_{EF} = 0$$

$$F_{EF} = -4.5 \text{ kN (Compression)}$$

$$\Sigma F_{y} = 0 \uparrow + \text{ ve}$$

$$-3 - F_{ED} = 0$$

$$F_{ED} = -3 \text{ kN}$$

$$F_{ED} = 3 \text{ kN (Compression)}$$

Isolating joint F

This joint has only one unknown member DF. Initially assuming the member to be in tension.

Applying COE
$$\sum F_x = 0 \rightarrow + \text{ ve}$$

$$4.5 - F_{DF} \cos 45 = 0$$

$$F_{DF} = 6.36 \text{ kN}$$

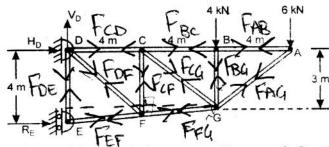
$$F_{DF} = 6.36 \text{ kN (Tension)}$$

$$F_{DF} = 6.36 \text{ kN (Tension)}$$

Finally tabulating the results.

Member	Force (kN)	Nature
AB	6.36	Tension
AC	6.5	Compression
BD	6.5	Tension
BC	4.5	Compression
CD	2.5	Compression
CE	4.5	Compression
EF	4.5	Compression
ED	3	Compression
DF	• 6.36	Tension

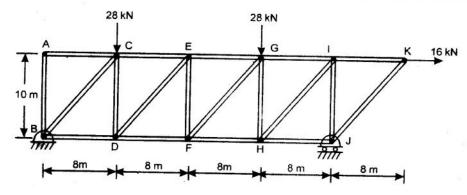
Ex. 5.2 For the truss shown find the forces in all the members.



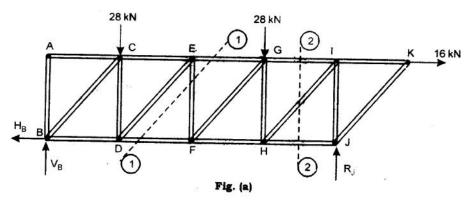
Solution: This truss is special since it has a *cantilever end*. Cantilever end is a joint having only two members and is externally unsupported. Analysis of a truss having a cantilever end need not begin from finding the reactions. One can start solving from the cantilever end.

The cantilever end of the given truss is joint A.

Ex. 5.3 For the truss shown find forces in members EF and GI by method of sections.



Solution: Figure (a) below shows the FBD of the truss.



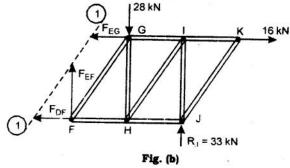
Applying COE to the entire truss

To find force in member EF

Cutting the truss by taking section (1) - (1) as shown in figure (a), the FBD of the R.H.S part is shown in figure (b)

Applying COE

$$\begin{split} & \sum F_y = 0 \\ & F_{EF} - 28 + 33 = 0 \\ & F_{EF} = -5 \text{ kN} \\ & F_{EF} = 5 \text{ kN (Compression)} \end{split}$$

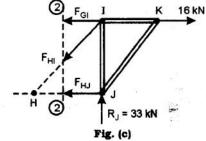


To find force in member GI

Cutting the truss by taking section (2) - (2) as shown in figure (a). The FBD of the R.H.S part is shown in figure (c)

Applying COE

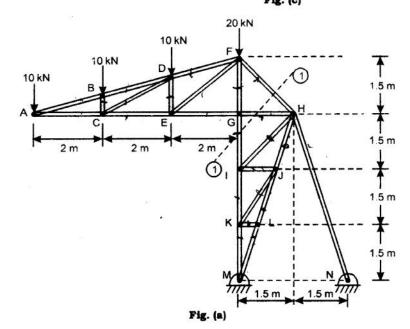
$$\sum M_H = 0$$
 $+ ve$
 $F_{GI} \times 10 + 33 \times 8 - 16 \times 10 = 0$
 $F_{GI} = -10.4 \text{ kN}$
 $F_{GI} = 10.4 \text{ kN}$ (Compression) .. **Ans.**



Ex. 5.4 Find the force in members FH, GI and GH of the stadium truss.

Solution: To find force in FH, GI and GH, let us take a cutting section (1)–(1) passing through them as shown in figure (a).

Let us take the L.H.S. part of the truss. This will avoid finding hinge reactions. The F.B.D of L.H.S part is shown in figure (b).



Applying COE

$$\sum_{i=1}^{n} M_{C} = 0 + ve$$

$$10 \times 6 + 10 \times 4 + 10 \times 2$$

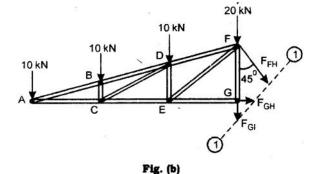
$$- (F_{FH} \sin 45) \times 1.5 = 0$$

$$F_{FH} = 113.1 \text{ kN}$$

$$F_{FH} = 113.1 \text{ kN} \text{ (Tension)} \qquad \text{Ans.}$$

$$\Sigma F_x = 0 \rightarrow + ve$$

 $F_{FH} \sin 45 + F_{GH} = 0$
113.1 $\sin 45 + F_{GH} = 0$
 $F_{GH} = -80 \text{ kN}$
 $F_{GH} = 80 \text{ kN}$ (Compression) Ans.



$$\begin{array}{l} \sum F_y = 0 \uparrow + ve \\ -F_{FH}\cos 45 - F_{GI} - 10 - 10 - 10 - 20 = 0 \\ -113.1\cos 45 - F_{GI} - 50 = 0 \\ F_{GI} = -130 \text{ kN} \\ F_{GI} = 130 \text{ kN (Compression)} \dots \text{.....Ans.} \end{array}$$

5.7 Special Cases

There are certain special cases which if identified and used lead to quicker solution. These special cases are discussed below.

Case 1: "If three members meet at a joint of which two are collinear, and there is no load at the joint, then the third member is a zero force member and the collinear members have the same force in magnitude and nature".

Fig. 5.2 (a) shows a joint J formed by three members AJ, BJ and CJ. Member AJ is collinear with BJ and there is no load at the joint, then by special *Case 1* we have,

$$F_{CJ} = 0$$
 and
$$F_{AJ} = F_{BJ}$$

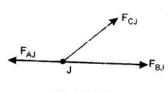


Fig. 5.2 (a)

Variant to which Case 1 can be applied

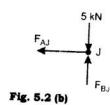
Joint J shows two members AJ and BJ and a load of 5 kN collinear with

We can apply the special Case 1, taking the 5 kN load as a member having a force of 5 kN Compression.

Now the conditions of special Case 1 have been satisfied. We can therefore say,

$$F_{AJ} = 0$$

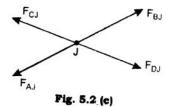
and $F_{BJ} = 5 \text{ kN (Comp.)}$



Case 2: "If four members meet at a joint, forming two pairs of collinear members, and there is no load at the joint, then the collinear members have the same force in magnitude and nature".

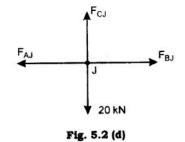
Fig. 5.2 (c) shows a joint formed by four members AJ, BJ, CJ and DJ. Member AJ is collinear with BJ and member CJ is collinear with DJ. Also there is no load at joint J. We can therefore say by special Case 2.

$$F_{AJ} = F_{BJ}$$
and
$$F_{CJ} = F_{DJ}$$



Variant to which Case 2 can be applied

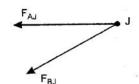
Fig. 5.2 (d) shows a joint formed by three members AJ, BJ and CJ. Also a load of 20 kN acts on it. Therefore the requirements of Case 2 are not being satisfied. If we assume the load to be a member having a force of 20 kN Tension, the condition of special Case 2 gets satisfied. We therefore can say,



 $F_{AJ} = F_{BJ}$ $F_{CJ} = 20 \text{ kN (Tension)}$

"If two members meet at a joint and the joint is unsupported and Case 3: unloaded, then both the members are zero force members".

Figure 5.2 (e) shows a joint formed by two members AJ and BJ. The joint J is unsupported and also no load acts on



We can therefore say by special Case 3

$$F_{AJ} = 0$$

and

 $F_{RI} = 0$

Fig. 5.2 (e)

Statically Determinate and Statically Indeterminate Truss 5.8

Statically Determinate Truss

A truss in which we can find the forces in all the members of the truss by applying the three conditions of equilibrium is known as Statically Determinate Truss. They are also referred to as a Perfect Truss.

All the trusses which we have solved were statically determinate. For a truss to be statically determinate, the following condition has to be satisfied.

$$m = 2j - r$$

Here

m = number of members

j = number of joints

r = number of reactions

Fig. 5.3 shows examples of Statically Determinate Trusses.

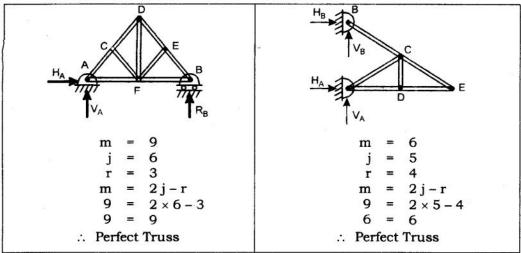


Fig. 5.3

Statically Indeterminate Truss

A truss in which we cannot find the forces in all the members of the truss using conditions of equilibrium is known as a *Statically Indeterminate Truss*. They are also referred to as *Imperfect Truss* and do not satisfy the relation

$$m = 2 j - r$$
.

They are of two types

- a) Redundant or Over Rigid Truss where m > 2 j r.
- b) Deficient Truss where m < 2j r

Figure 5.4 shows examples of Statically Indeterminate Trusses

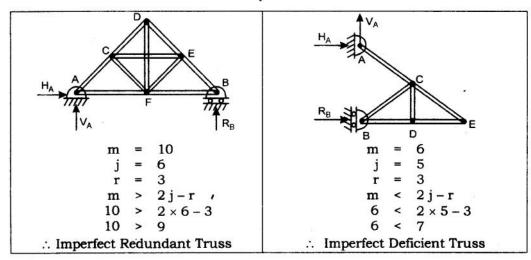
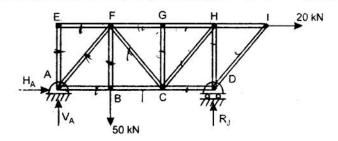


Fig. 5.4

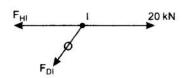
Ex. 5.5 Without calculation find by inspection, forces in many members possible.



Solution:

Applying special Case 1 to joint I, we have $F_{HI} = 20 \text{ kN (Tension)} \dots Ans.$

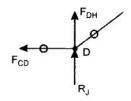
also $\mathbf{F}_{\text{DI}} = \mathbf{0}$ Ans.



Applying special Case1 to joint D

 $F_{CD} = 0$

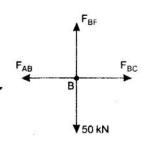
also $\mathbf{F}_{\mathrm{DH}} = \mathbf{R}_{\mathrm{J}}$



Applying special Case 2 to joint B

 $F_{BF} = 50 \text{ kN (Tension)} \dots \text{Ans.}$

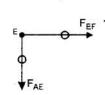
also $F_{AB} = F_{BC}$



Applying special Case 3 to joint E

 $F_{AE} = 0$ Ans.

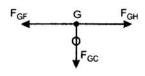
 $F_{EF} = 0$ also Ans.



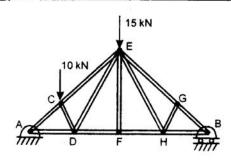
Applying special Case 1 to joint G

 $F_{GC} = 0$ Ans.

 $\mathbf{F}_{GF} = \mathbf{F}_{GH}$ also



Ex. 5.6 Identify zero force members for the truss shown.



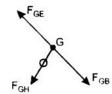
Solution:

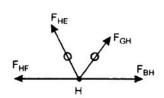
Applying special Case 1 to joint G

$$F_{GH} = 0$$
 Ans.

Applying special Case 1 to joint H

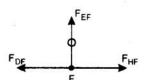
$$F_{HE} = 0$$
 Ans.





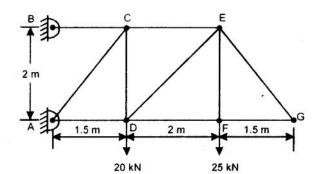
Applying special Case 1 to joint F

$$F_{EF} = 0$$
 Ans.



Ex. 5.7 For the pin joined truss

- a. Check if the truss is perfect or imperfect
- b. Find the support reactions
- c. Find forces in all members of
- d. Check force in members CE, DE and DF by method of sections.



Solution:

a. To check if truss is perfect or imperfect

The truss has 10 members, 7 joints and 4 support reactions

For a perfect truss
$$m = 2j - r$$

$$10 = 2 \times 7 - 4$$

$$10 = 10$$

the truss is perfect

..... Ans.

2 m

20 kN

58.75 kN

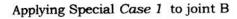
b. To find support reactions

Applying COE to the entire truss

$$\Sigma F_{x} = 0 \rightarrow + ve$$

$$H_{A} - 58.75 = 0$$

$$\therefore H_{A} = 58.75 \text{ kN} \rightarrow$$

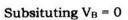


$$\begin{array}{c} V_B = 0 \\ \text{also} & F_{BC} = 58.75 \text{ kN (Tension)} \end{array}$$

Applying COE to entire truss

$$\sum F_y = 0 \uparrow + ve$$

$$V_B + V_A - 20 - 25 = 0$$



$$V_A = 45 \text{ kN}$$

$$= 45 \text{ kN} \uparrow$$

∴ Recation at A =
$$H_A$$
 = 58.75 kN \rightarrow , V_A = 45 kN \uparrow

...... Ans.

1.5 m

25 kN

 \mathbf{F}_{BC}

$$\therefore$$
 Recation at B = H_B = 58.75 kN \leftarrow , V_A = 0

..... Ans.

c. To find the forces in all members of truss

Isolating joint G

Applying Special Case 3 to joint G

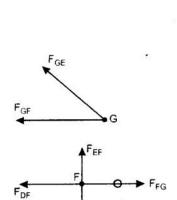
$$\mathbf{F}_{\mathrm{GE}} = \mathbf{F}_{\mathrm{GF}} = \mathbf{0}$$

Isolating joint F

Applying Special Case 2 to joint F

$$\therefore$$
 F_{EF} = 25 kN (Tension)

also
$$F_{DF} = 0$$



▼25 kN

Isolating joint E

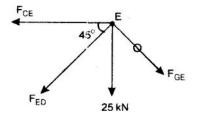
This joint has two unknown members CE and ED. Initially assuming them to be in tension

Applying COE

$$\begin{array}{l} \sum F_y = 0 \ \uparrow + ve \\ -25 - F_{ED} \sin 45 = 0 \\ F_{ED} = -35.35 \ kN \end{array}$$

$$\therefore \qquad F_{ED} = 35.35 \ kN \ (compress.$$

$$\begin{array}{lll} \Sigma F_x = 0 \to + ve \\ - F_{CE} - F_{ED} \cos 45 = 0 \\ - F_{CE} - (-35.35) \cos 45 = 0 \\ F_{CE} = 25 \text{ kN} \\ \therefore & F_{CE} = 25 \text{ kN (Tension)} \end{array}$$



Isolating joint D

This joint has two unknown members AD and CD. Initially assuming them to be in tension.

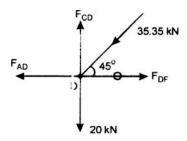
Applying COE

$$\sum F_y = 0 \uparrow + ve$$

$$F_{CD} - 35.35 \sin 45 - 20 = 0$$

$$F_{CD} = 45 \text{ kN}$$

$$\therefore F_{CD} = 45 \text{ kN (Tension)}$$



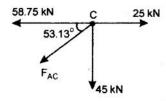
Isolating joint C

 F_{BC} = 58.75 kN (T) has been found out while finding support reactions. Member AC is the only unknown member. Assuming the member to be in tension.

Applying COE

$$\Sigma F_x = 0 \rightarrow + ve$$
- 58.75 + 25 - F_{AC} cos 53.13 = 0

 $F_{AC} = -56.25$ kN
∴ $F_{AC} = 56.25$ kN (Compression)

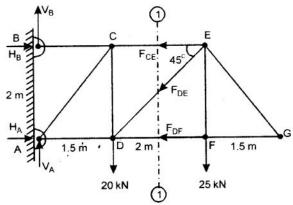


Finally Tabulating the results

Member	Force (kN)	Nature
GE	0	_
GF	0	_
DF	0	_
EF	25	Tension
ED	35.35	Compression
CE	25	Tension
AD	25	Compression
CD	45	Tension
BC	58.75	Tension
AC	56.25	Compression

d. To check forces in members CE, DE and DF by method of section

Taking section (1) - (1) and applying COE to RHS of the truss. Three unknown members CE, DE and DF have been cut. Initially assuming them to be in tension



$$\begin{split} \sum F_{y} &= 0 \uparrow + ve \\ &- F_{DE} \sin 45 - 25 = 0 \\ &F_{DE} &= -35.35 \text{ kN} \\ \therefore &F_{DE} &= 35.35 \text{ kN (compression)} \end{split}$$
 Ans.
$$\sum M_{E} &= 0 \qquad + ve \\ &- F_{DF} \times 2 = 0 \\ \therefore &F_{DF} &= 0 \qquad \text{Ans.} \end{split}$$

$$\begin{array}{l} \sum F_x = 0 \ \ \, \rightarrow \ \ \, + \ \, ve \\ - \ \, F_{CE} - F_{DE} \cos \, 45 - F_{DF} = 0 \\ - \ \, F_{CE} - (-\ \, 35.35) \cos \, 45 - 0 = 0 \\ F_{CE} = 25 \ \, kN \\ \therefore \qquad F_{DE} = 25 \ \, kN \ \, (Tension) \end{array}$$

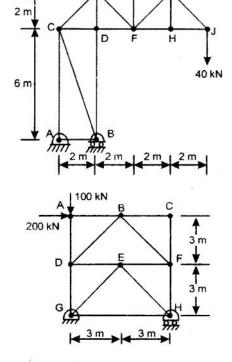
...... Ans.

7

P9. For the pin-joined truss loaded as shown, find

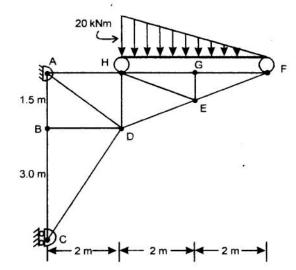
- a. All the reactions at A and B
- b. Forces in members EC, ED and DF by method of sections.
- Identify all the zero force members giving reasoning for each member.
- d. Axial forces in remaining members by method of joints.

P10. Find forces in members DG and FH by method of sections.

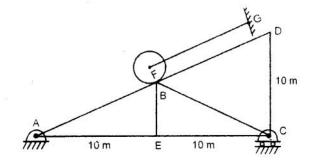


P11. For the truss shown find,

- a. Support reaction
- b. Solve joints F and G by method of joints.
- c. Find AH and AD by method of sections.



P12. A sphere of weight 1000 N rests on joint B. It is kept from rolling down the plane by a cable FG. Cable FG is parallel to portion ABD of the truss. Find support reactions and forces in all members of the truss.



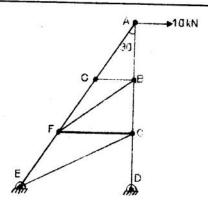
Excercise 5.3

Theory Questions

- Q.1 Define Truss.
- Q.2 What are the applications of truss.
- Q.3 What is meant by analysis of truss. Explain in brief the different methods of analysis.
- Q.4 Explain the difference between method of joints and method of sections used in the solution of pin-jointed frames.
- Q.5 What is a Perfect Truss and an Imperfect Truss?
- Q.6 What do you understand by "Redundant Truss"?
- Q.7 Explain: a) Deficient frame b) Over rigid frame
- Q.8 State assumptions made in the analysis of plane truss.
- Q.9 Define Determinate and Indeterminate structures.



P10. Find forces in all the members of the truss.



5.6. Method of Sections

In this method the entire truss is cut and separated into two parts. After separation all the three COE are applied to any one part of the truss and thus force in the members is found out. This method is based on the principle, "If the truss is in equilibrium, an isolated part of the truss will also be in equilibrium".

Method of Sections offers immediate solution to any member desired, unlike method of joints where we have to solve various joints one by one to get to the desired member. However method of sections is preferred for few members analysis, while method of joints is suited when the whole truss is to be analysed.

The following steps are to be adopted while solving the truss by method of sections.

Step 1: Tick mark the members which have to be analysed.

Step 2: Take a cutting section passing through the tick marked members (not necessary through all the tick marked members) such that not more than three unknown members are cut. Also see that at least two joints are present in each part.

Step 3: Select any one of the two parts and isolate it from the rest of the truss.

Step 4: Assume that the unknown members carry tensile force. Now apply all the three COE viz. $\Sigma F_x = 0$, $\Sigma F_y = 0$ and $\Sigma M = 0$ and solve to get forces in the desired members. Moments are usually taken about a point where two unknown forces meet to find the third force.

Step 5: If the value obtained is negative it would imply that the assumption is incorrect and the member has compressive nature of the force.

Step 6: More than one cutting section may be required to be taken for finding the forces in the desired members.